Bi-Directional Coupling Between Two Coupled Transmission Lines

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***Abstract*—Coupled mode theory including backward coupling is employed to investigate the bi-directional coupling mechanism in two symmetrical coupled microstrip transmission lines. Analytical scattering parameter expressions for the case of matched terminations are presented. Reflections and near-end crosstalk is attributed to the combined effects of distributed backward coupling and interference between discrete reflections caused by termination mismatches at two ends.**

***Index Terms*—Coupled mode analysis, coupled transmission lines, scattering parameters.**

I. INTRODUCTION

COUPLING in two parallel transmission lines is a very common and yet important phenomenon in microwave and optical wave circuits and devices. Many methods have been developed to study such coupling. Among them, coupled mode theory is widely used because of its mathematical simplicity and physical intuitivity. When employing the coupled mode approach, researchers working in microwave and optical wave have different starting points. Coupled telegraphist's equations for voltages and currents are usually presumed in microwave regime, where the coupling is described by mutual capacitance and inductance per unit length. These voltages and currents can be expressed as the forward and backward waves, which will lead to coupled-mode formulations [1], [2]. In analyzing coupled optical waveguides, the total field of the structure is expressed as the superposition of the individual waveguide's eigen modes, and then perturbation theory, reciprocity theorem or variational principle are applied to derive coupled mode equations [3]–[7]. Non-orthogonality between modes belonging to different individual waveguides can also be included in the equations [4]–[7].

Yasumoto extended the nonorthogonal coupled mode theory to the analysis of multilayered and multiconductor transmission lines by using a generalized reciprocity theorem [8], where the coupling coefficients are expressed as the overlapping integrals of fields and currents of individual lines. The theory was applied to calculate the parameters of the compound structure, including the propagation constants and impedance matrix [9].

The inclusion of backward coupling in microwave regime is natural because the voltages and currents are the sum of forward and backward waves. For uniform coupled optical waveguides, the backward coupling is often ignored because the backward coupling is usually small [10]. In coupled microwave transmission lines, the backward coupling is sometimes even stronger than the forward coupling.

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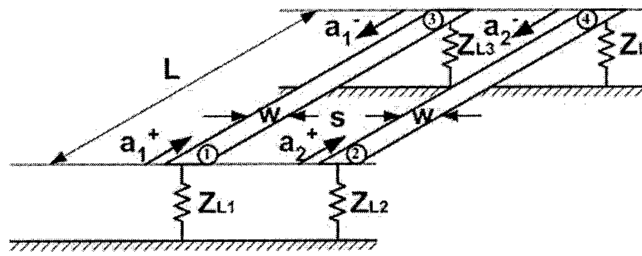

Fig. 1. Configuration of two symmetric coupled microstrip lines with terminations.

In this letter, we employ the coupled mode theory including backward coupling in [11] to investigate the bi-directional coupling in symmetric coupled transmission lines. In Section II, we derive closed-form expressions for the scattering parameters from coupled mode equations, especially for the case with matched terminations. In Section III, we discuss the mechanism of backward coupling, physical interpretations of the derived expressions, and also the effect of nonothogonality.

II. FORMULATION

The configuration of two symmetrical coupled microstrip lines is shown in Fig. 1. We assume that the two lines are lossless for simplicity.

Noticing the relationship between forward and backward modes, we can approximately express the electromagnetic fields and currents of the compound structure with the modes of individual lines when the coupling between the two lines is weak:

$$\mathbf{E}_t(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{e}_{\nu t}(x,y)+a_\nu^-(z)\mathbf{e}_{\nu t}(x,y)\right] \quad (1a)$$

$$\mathbf{E}_z(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{e}_{\nu t}(x,y)-a_\nu^-(z)\mathbf{e}_{\nu t}(x,y)\right] \quad (1b)$$

$$\mathbf{H}_t(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{h}_{\nu t}(x,y)-a_\nu^-(z)\mathbf{h}_{\nu t}(x,y)\right] \quad (1c)$$

$$\mathbf{H}_z(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{h}_{\nu z}(x,y)+a_\nu^-(z)\mathbf{h}_{\nu z}(x,y)\right] \quad (1d)$$

$$\mathbf{J}_t(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{j}_{\nu t}(x,y)+a_\nu^-(z)\mathbf{j}_{\nu t}(x,y)\right] \quad (1e)$$

$$\mathbf{J}_z(x,y,z) \approx \sum_{\nu=1}^{2} \left[a_\nu^+(z)\mathbf{j}_{\nu z}(x,y)-a_\nu^-(z)\mathbf{j}_{\nu z}(x,y)\right] \quad (1f)$$

where $a_1^+, a_2^+, a_1^-, a_2^-$ are the forward and backward mode magnitudes belonging to two individual lines. Applying the generalized reciprocity relation to a z-translational invariant system [5], [8], [11], we can derive the following coupled mode equation that describes the power transfer between the two lines:

$$\begin{bmatrix} 1 & N & 0 & 0 \\ N & 1 & 0 & 0 \\ 0 & 0 & 1 & N \\ 0 & 0 & N & 1 \end{bmatrix} \frac{d}{dz} \begin{bmatrix} a_1^+ \\ a_2^+ \\ a_1^- \\ a_2^- \end{bmatrix} = -j \begin{bmatrix} \beta_0 & \beta_0 N + \kappa^+ & 0 & \kappa^- \\ \beta_0 N + \kappa^+ & \beta_0 & \kappa^- & 0 \\ 0 & -\kappa^- & -\beta_0 & -\beta_0 N - \kappa^+ \\ -\kappa^- & 0 & -\beta_0 N - \kappa^+ & -\beta_0 \end{bmatrix} \times \begin{bmatrix} a_1^+ \\ a_2^+ \\ a_1^- \\ a_2^- \end{bmatrix} \tag{2a}$$

where β_0 is the propagation constant of a single microstrip line, N is the parameter originated from the nonothogonality between individual lines' eigen modes, κ^+ and κ^- are coupling coefficients between the co-propagating and counter-propagating modes belonging to different lines. N, κ^+ and κ^- can be expressed by overlap integrals of individual lines' fields and currents as follows:

$$N = \frac{1}{2} \int_S \mathbf{e}_{1t}(x,y) \times \mathbf{h}_{2t}(x,y) \cdot \mathbf{i}_z dxdy \tag{2b}$$

$$\kappa^+ = -\frac{j}{4} \int_{S_2} [\mathbf{e}_{1t}(x,y) \cdot \mathbf{j}_{2t}(x,y) - e_{1z}(x,y) j_{2z}(x,y)] dxdy \tag{2c}$$

$$\kappa^- = -\frac{j}{4} \int_{S_2} [\mathbf{e}_{1t}(x,y) \cdot \mathbf{j}_{2t}(x,y) + e_{1z}(x,y) j_{2z}(x,y)] dxdy \tag{2d}$$

where S is the cross-section of the entire structure, S_2 is the surface of the second strip. The fields and currents can be obtained by any 2D field solver [8], [11]. The value of $\kappa^\pm/\beta_0$ is dependant upon the distance between two lines. Normally, they are of the order of 10^{-1} or less. For other types of transmission lines, such as coupled optical waveguides, the following derivation and discussions will also be applicable, though these parameters may have different expressions.

When ignoring the nonorthogonal parameter N, which is often small for weak coupling, the eigen-values of the coupling matrix in (2a) are

$$\{\beta_e, \beta_o, -\beta_e, -\beta_o\} \tag{3a}$$

where

$$\beta_e = \sqrt{(\beta_0 + \kappa^+ + \kappa^-)(\beta_0 + \kappa^+ - \kappa^-)} \tag{3b}$$

$$\beta_o = \sqrt{(\beta_0 - \kappa^+ - \kappa^-)(\beta_0 - \kappa^+ + \kappa^-)} \tag{3c}$$

Subscriptions "e" and "o" represent even and odd modes, respectively.

The corresponding eigen-vectors (not normalized) are:

$$\{\mathbf{Q}_e^+, \mathbf{Q}_o^+, \mathbf{Q}_e^-, \mathbf{Q}_o^-\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ \alpha_e \\ \alpha_e \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -\alpha_o \\ \alpha_o \end{bmatrix}, \begin{bmatrix} \alpha_e \\ \alpha_e \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} \alpha_o \\ -\alpha_o \\ -1 \\ 1 \end{bmatrix} \right\} \tag{3d}$$

where

$$\alpha_e = \frac{1}{2} \frac{-\kappa^-}{\beta_0 + \kappa^+} \tag{3e}$$

$$\alpha_o = \frac{1}{2} \frac{-\kappa^-}{\beta_0 - \kappa^+} \tag{3f}$$

The solution to coupled mode equations can then be expressed as:

$$\begin{bmatrix} a_1^+ \\ a_2^+ \\ a_1^- \\ a_2^- \end{bmatrix} = A_e^+ \mathbf{Q}_e^+ e^{-j\beta_e z} + A_o^+ \mathbf{Q}_o^+ e^{-j\beta_o z} + A_e^- \mathbf{Q}_e^- e^{j\beta_e z} + A_o^- \mathbf{Q}_o^- e^{j\beta_o z} \tag{4}$$

If we know the boundary conditions of the four waves along the two lines, we can obtain the values of A_e^+, A_o^+, A_e^-, A_o^- and then derive the scattering parameters of the coupled structure. When we have input only from port 1 with $a_1^+(0) = 1$, for the case of matched terminations, we obtain:

$$A_e^+ = \frac{1}{2(1 - \alpha_e^2 e^{-j2\beta_e L})} \tag{5a}$$

$$A_o^+ = \frac{1}{2(1 - \alpha_o^2 e^{-j2\beta_o L})} \tag{5b}$$

$$A_e^- = \frac{-\alpha_e e^{-j2\beta_e L}}{2(1 - \alpha_e^2 e^{-j2\beta_e L})} \tag{5c}$$

$$A_o^- = \frac{-\alpha_o e^{-j2\beta_o L}}{2(1 - \alpha_o^2 e^{-j2\beta_o L})} \tag{5d}$$

By neglecting the fourth-order terms involving α_e and α_o, we can derive the magnitudes of the scattering parameters:

$$|S_{11}|^2 \approx [\alpha_e \sin(\beta_e L) + \alpha_o \sin(\beta_o L)]^2 - 4\alpha_e \alpha_o \sin(\beta_e L) \sin(\beta_o L) \cos^2\left(\frac{\beta_e - \beta_o}{2} L\right) \tag{6a}$$

$$|S_{21}|^2 \approx [\alpha_e \sin(\beta_e L) + \alpha_o \sin(\beta_o L)]^2 - 4\alpha_e \alpha_o \sin(\beta_e L) \sin(\beta_o L) \sin^2\left(\frac{\beta_e - \beta_o}{2} L\right) \tag{6b}$$

$$|S_{31}|^2 \approx \cos^2\left(\frac{\beta_e - \beta_o}{2} L\right) - \alpha_e^2 \sin(\beta_e L) \times [\sin(\beta_e L) + \sin(2\beta_e - \beta_o)L] - \alpha_o^2 \sin(\beta_o L) \times [\sin(\beta_o L) + \sin(2\beta_o - \beta_e)L] \tag{6c}$$

$$|S_{41}|^2 \approx \sin^2\left(\frac{\beta_e - \beta_o}{2} L\right) - \alpha_e^2 \sin(\beta_e L) \times [\sin(\beta_e L) - \sin(2\beta_e - \beta_o)L] - \alpha_o^2 \sin(\beta_o L) \times [\sin(\beta_o L) - \sin(2\beta_o - \beta_e)L] \tag{6d}$$

For a more general termination condition, the expressions of scattering parameters can also be derived in the similar way, though they may be more complicated.

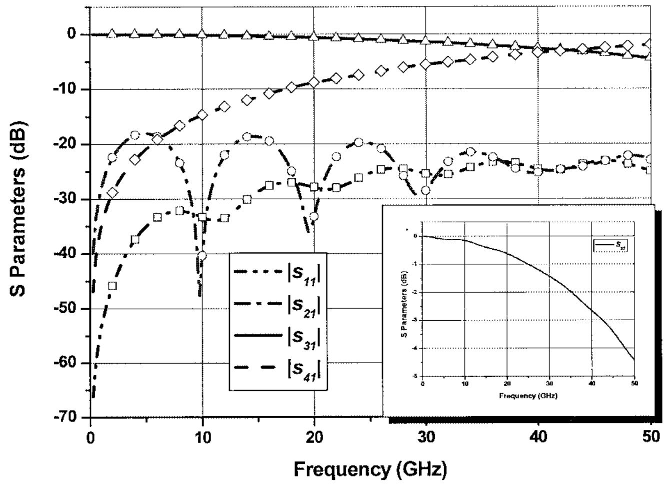

Fig. 2. Scattering parameters of two symmetric coupled microstrip with matched terminations. Results of lines are from (6a)–(6d), while the symbols are from (4).

III. Discussions

The formulation presented in Section II shows that even in the case of matched terminations, there are still backward reflections. This is easy to understand from the view point of even-odd mode analysis , where the matched terminations are actually not matched to the even and odd modes of the coupled structure, which will result in reflections at both ends. From the coupled mode theory viewpoint, if the backward waves are not included from the beginning, there is no information to predict the backward coupling in matched terminations [8], [11]. In general, the reflections $|S_{11}|^2$ and near-end crosstalk $|S_{21}|^2$ have two sources: one is the distributed backward coupling as in the case of matched terminations, the other is the interference of reflections from the unmatched terminations at both ends. This is also true for the forward coupling.

We show an example in Fig. 2 for the scattering parameters of two coupled microstrip lines with nearly matched terminations to illustrate the coupling mechanism that the derived analytical expressions can predict. Comparison of the solutions from (6a)–(6d) with exact solution (4) is also given in the figure. Parameters used in expressions (6a)–(6d) and (4): $\beta_0/k_0 = 2.6$, $L = 6\ mm$, $|\kappa^+|/\beta_0 = 0.0556$, $|\kappa^-|/\beta_0 = 0.139$.

The power flows out of through-and cross-arms follow the relations of $\cos^2(((\beta_e - \beta_o)/2)L)$ and $\sin^2(((\beta_e - \beta_o)/2)L)$ with small fluctuations. These fluctuations, caused by the power transfer to the backward waves, affect the output power of through-arm more than that of the cross-arm, which is highlighted in the inserted window in Fig. 2. The magnitude of the far-end coupling, $|S_{41}|^2$ follows a quasiperiodic variation with respect to the frequency because the value of $(\beta_e - \beta_o)$ is nearly proportional to the operating frequency. Expressions of $|S_{11}|^2$ and $|S_{21}|^2$ predict that they also experience quasiperiodic characteristics, with a period much shorter than those of $|S_{31}|^2$ and $|S_{41}|^2$. We can further conclude that maximum values of $|S_{21}|^2$ are always higher than or equal to those of $|S_{11}|^2$, physically because $|S_{21}|^2$ is mainly caused by the backward coupling from the through-arm, while $|S_{11}|^2$ from the cross-arm. The integral of power along the through line is always bigger than that of the cross line. Both the coupling coefficients and coupling length will affect the far-end crosstalk, but the maximums of the near-end crosstalk are only affected by the coupling coefficient.

It can be easily verified that expressions in (6) satisfy the power conservation principle:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1 \quad (7)$$

Finally, it should be mentioned that the above formulation is still valid for the case including the nonorthogonality effect, while β_e, β_o, α_e, α_o should be replaced by the following expressions.

$$\beta_e = \sqrt{\left(\beta_0 + \frac{\kappa^+ - \kappa^-}{1+N}\right)\left(\beta_0 + \frac{\kappa^+ + \kappa^-}{1+N}\right)} \quad (8a)$$

$$\beta_o = \sqrt{\left(\beta_0 - \frac{\kappa^+ - \kappa^-}{1-N}\right)\left(\beta_0 - \frac{\kappa^+ + \kappa^-}{1-N}\right)} \quad (8b)$$

$$\alpha_e \approx \frac{1}{2}\frac{-\kappa^-}{\beta_0(1+N) + \kappa^+} \quad (8c)$$

$$\alpha_o \approx \frac{1}{2}\frac{-\kappa^-}{\beta_0(1-N) - \kappa^+} \quad (8d)$$

When the strips are well separated, the nonorthogonal parameter is far less than 1 and thus can be omitted.

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